RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2614/1

Statistics 2

Thursday **9 JUNE 2005** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

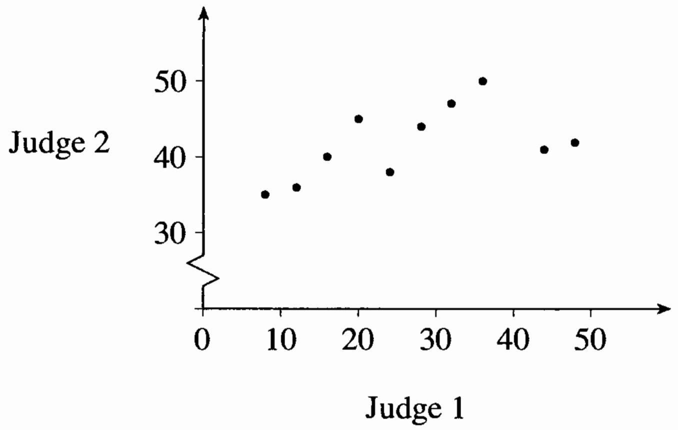
This question paper consists of 4 printed pages.

1 The directors of a large retail chain are investigating the performance of their shops. Two judges inspect the shops and give them marks out of 50.

The table shows the marks given to each of a random sample of 10 shops.

Shop	A	B	C	D	E	F	G	H	I	J
Judge 1	44	28	12	24	16	32	48	8	20	36
Judge 2	41	44	36	38	40	47	42	35	45	50

The data are illustrated on the scatter diagram below.



(i) Calculate Spearman's rank correlation coefficient for the data. [5]

(ii) Hence examine whether, on the whole, the judges are in agreement, by means of a hypothesis test at the 5% significance level. [5]

(iii) A director asks whether, since the data are numerical, it would be better to perform a test based on the product-moment correlation coefficient. State an assumption required for the use of the product-moment correlation coefficient and comment on the director's question in the light of this assumption. [2]

(iv) The managers of shops G and J each claim that their shop has done best. Suggest how they arrive at their claims, and comment briefly. [3]

2 A clothing manufacturer makes men's trousers in three different lengths: *Long*, *Medium*, and *Short*. You may assume that the leg length of men can be modelled by a Normal distribution with mean 77 cm and standard deviation 3 cm. The *Medium* length trousers are suitable for men whose leg length is between 74.5 and 81.2 cm.

(i) Draw a sketch of this Normal distribution, showing the mean. Indicate clearly the area representing *Medium* trousers. [2]

(ii) Find the proportion of men for whom *Medium* length trousers would be suitable. [4]

(iii) Find the probability that exactly 14 out of 20 men wishing to purchase trousers from a retailer would find *Medium* length trousers suitable. State an assumption that is necessary for your calculations to be valid. [3]

Following complaints by a number of customers that the *Long* trousers are not long enough, the manufacturer introduces a new length, *Extra Long*, which is suitable for the 2% of men who have the longest legs.

(iv) Find the shortest leg length for which the new *Extra Long* trousers would be suitable. [3]

(v) Find the minimum value of n for which it is at least 90% certain that one or more of a group of n randomly selected men will require *Extra Long* trousers. [3]

3 A student keeps a record of the number of items of junk mail which she receives each day in the post. She does not include Sundays or public holidays when there is no mail delivery.

(i) Explain why a Poisson distribution might provide a suitable model for the number of items of junk mail which arrive on a particular day. [2]

(ii) The results recorded by the student over a randomly selected sample of 100 days are given below.

Number of items of junk mail	0	1	2	3	4	$\geqslant 5$
Number of days	29	31	25	11	4	0

Show that the mean of these data is 1.3 and calculate the variance. [3]

(iii) Do your calculations in part **(ii)** support the suggestion that a Poisson distribution is a suitable model? Explain your answer. [1]

(iv) Use suitable Poisson distributions to find the probability of

(A) receiving exactly 2 items of junk mail on a particular day,

(B) receiving more than 10 items of junk mail in a period of six days. [5]

(v) Use a suitable approximating distribution to calculate the probability of receiving at least 80 items of junk mail in 50 days of deliveries. [4]

4 In each round of a television quiz, a contestant is shown pictures of four locations in the world and, separately, the names of the four locations. The task is to match the names to the locations correctly. Since the locations are not well known, it may be assumed that a contestant guesses randomly. The random variable X represents the number of locations which a contestant names correctly.

The table shows the probability distribution of X.

r	0	1	2	3	4
$\mathrm{P}(X=r)$	$9k$	$8k$	$6k$	0	k

(i) (*A*) Explain why $\mathrm{P}(X = 3) = 0$.

(*B*) Calculate the value of k.

(*C*) Use a probability argument to justify the value of $\mathrm{P}(X = 4)$. [5]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [4]

(iii) A contestant is given a prize of £100 for each location which is named correctly. Write down the expectation and variance of the prize money won by a contestant in one round of the contest. [2]

A contestant who correctly names all four locations in the first round goes on to play five more rounds. In each of these five additional rounds, the prize money is increased to £1000 for every correctly named location. Emma takes part in the quiz and she correctly names all four locations in the first round.

(iv) Assuming that Emma continues to guess the locations randomly,

(*A*) find the probability that in exactly three of the five additional rounds, Emma correctly matches just one of the four locations,

(*B*) find Emma's expected total prize money over all six rounds. [4]

Mark Scheme 2614
June 2005

GENERAL INSTRUCTIONS

Marks in the mark scheme are explicitly designated as **M**, **A**, **B**, **E** or **G**.

M marks ("method") are for an attempt to use a correct method (not merely for stating the method).

A marks ("accuracy") are for accurate answers and can only be earned if corresponding **M** mark(s) have been earned. Candidates are expected to give answers to a sensible level of accuracy in the context of the problem in hand. The level of accuracy quoted in the mark scheme will sometimes deliberately be greater than is required, when this facilitates marking.

B marks are independent of all others. They are usually awarded for a single correct answer. Typically they are available for correct quotation of points such as 1.96 from tables.

E marks ("explanation") are for explanation and/or interpretation. These will frequently be sub divisible depending on the thoroughness of the candidate's answer.

G marks ("graph") are for completing a graph or diagram correctly.

- Insert part marks in **right-hand** margin in line with the mark scheme. For fully correct parts tick the answer. For partially complete parts indicate clearly in the body of the script where the marks have been gained or lost, in line with the mark scheme.

- Please indicate incorrect working by ringing or underlining as appropriate.

- Insert total in **right-hand** margin, ringed, at end of question, in line with the mark scheme.

- Numerical answers which are not exact should be given to at least the accuracy shown. Approximate answers to a greater accuracy *may* be condoned.

- Probabilities should be given as fractions, decimals or percentages.

- FOLLOW-THROUGH MARKING SHOULD NORMALLY BE USED WHEREVER POSSIBLE. There will, however, be an occasional designation of '**c.a.o.**' for "correct answer only".

- Full credit MUST be given when correct alternative methods of solution are used. If errors occur in such methods, the marks awarded should correspond as nearly as possible to equivalent work using the method in the mark scheme.

- The following notation should be used where applicable:

FT	Follow-through marking
BOD	Benefit of doubt
ISW	Ignore subsequent working

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Question 1

(i)	*Shop* / *Judge* 1 / *Judge* 2 / *Rank* 1 / *Rank* 2 / *d* / d^2: A 44 41 2 6 -4 16 B 28 44 5 4 1 1 C 12 36 9 9 0 0 D 24 38 6 8 -2 4 E 16 40 8 7 1 1 F 32 47 4 2 2 4 G 48 42 1 5 -4 16 H 8 35 10 10 0 0 I 20 45 7 3 4 16 J 36 50 3 1 2 4 $r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times 62}{10\times 99} = 1 - \frac{372}{990} = 1 - 0.3758$ $= 0.62$ (to 2 s.f.) [*or* 0.624 to 3 s.f.]	B2 for ranks (B1 for ≤ 3 errors) B1 for d^2 f.t. their ranks M1 for r_s A1 f.t. for $\lvert r_s\rvert < 1$ *Allow use of pmcc on ranks*	**5**
(ii)	H_0: 'Independence' or $\rho = 0$; H_1: Positive 'association' or $\rho > 0$ [one tailed test] Looking for positive association: critical value at 5% level is 0.5636 Since 0.624 > 0.5636, there is sufficient evidence to reject H_0, i.e. there is sufficient evidence to conclude that there is positive association between the judges' marks.	B1 for H_0; B1 for H_1 B1 for ± 0.5636 [f.t. from their H_1] M1 for comparison with c.v., provided $\lvert r_s\rvert < 1$ A1 for conclusion in context	**5**
(iii)	A test based on the pmcc requires a *bivariate Normal* parent population. Sensible comment for/against the use of pmcc with a reason, relating to *ellipticity* of the scatter diagram.	B1 B1 for conclusion with reason	**2**
(iv)	• Shop G is best according to judge 1 and shop J is best according to judge 2. • Shop J is the best overall, (because) • Shop J has the best overall pair of ranks (first and third) *Or* • Shop G has the highest total score of 90. • Shop J has the best overall pair of ranks (first and third) • Basing this judgement on position may be fairer, since the marks are more subjective than the rankings, (particularly since there is such a large discrepancy in the dispersion of	E1 E1 E1 *Or* E1 E1 E1	

	the marks awarded by each judge).		**3**
			15

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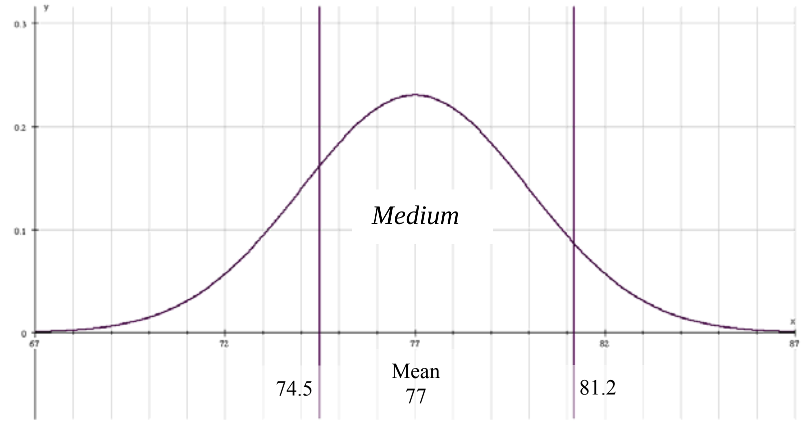
Question 2

(i)		G1 for shape and mean = 77 G1 for 'Medium' and limits 74.5 and 81.2	**2**
(ii)	$P(74.5 < X < 81.2) = P\left(\frac{74.5-77}{3} < Z < \frac{81.2-77}{3}\right)$ $= P(-0.8333 < Z < 1.4)$ $= 0.9192 - (1 - 0.7976)$ $= 0.7168$ *or* 0.717 (to 3 s.f.) *or* 0.72 (to 2 s.f.) so the proportion is 72% (to 2 s.f.)	M1 for $74.5 < X < 81.2$ M1 for standardizing M1 for prob. calc. A1	**4**
(iii)	P(14 out of 20 are medium) $= \binom{20}{14} \times 0.7168^{14} \times 0.2832^{6}$ $= 0.189$ (to 3 s.f.) *or* $\binom{20}{14} \times 0.72 \times 0.28 = 0.188$ (to 3 s.f.) The 20 men must form an independent random sample.	M1 for $\binom{20}{14} \times p^{14} \times q^{6}$ [where $q = 1 - p$] A1 B1 for 'random' *or* 'independent'	**3**
(iv)	From tables $\Phi^{-1}(0.98) = 2.054$ $\frac{x-77}{3} = 2.054$ $\Rightarrow x = 77 + 3 \times 2.054$ $\Rightarrow x = 83.2$ cm	B1 for 2.054 seen M1 for equation in x with sensible positive z-value A1 **cao**	**3**
(v)	$1 - 0.98^{n} > 0.9$ *Or* $1 - e^{-0.02n} > 0.9$ $\Rightarrow 0.98^{n} < 0.1$; $\Rightarrow e^{-0.02n} < 0.1$ $\Rightarrow n \log 0.98 < \log 0.1$; $\Rightarrow -0.02n < \ln 0.1$ $\Rightarrow n > \log 0.1 / \log 0.98$; $\Rightarrow n > \ln 0.1 / (-0.02)$ $\Rightarrow n > 113.974$; $\Rightarrow n > 115.129$ $\Rightarrow$ Min. value of n is 114 ; $\Rightarrow$ Min. value of n is 116	B1 for inequality M1 for attempt to solve by logs (including Poisson approximation) or by trial and improvement A1 **cao**	**3**
			15

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Question 3

(i)	*Uniform (average) rate of occurrence* Junk mail is likely to arrive *randomly* and/or *independently*	E1 for suitable reason E1 for suitable reason in context	**2**
(ii)	Mean $= \dfrac{\Sigma xf}{n} = \dfrac{31+50+33+16}{100} = \dfrac{130}{100} = 1.3$ Variance $= \dfrac{1}{n}\left(\Sigma x^2 f - \dfrac{(\Sigma xf)^2}{n}\right)$ $= \dfrac{1}{100}\left(294 - \dfrac{130^2}{100}\right) = 1.25$ NB *Answer is 1.263 with divisor* n – *1* Or Variance $= \dfrac{\Sigma x^2 f}{n} - \bar{x}^2 = \dfrac{31+100+99+64}{100} - 1.3^2$ $= \dfrac{294}{100} - 1.3^2 = 1.25$	B1 for mean M1 for calculation A1 *Or* M1 for calculation A1	**3**
(iii)	Yes, since mean is close to variance	B1	**1**
(iv)	(*A*) $P(X=2) = e^{-1.3}\dfrac{1.3^2}{2!}$ $= 0.230$ (to 3 s.f.) $= 0.23$ (to 2 s.f.) (*B*) $\lambda = 6 \times 1.3 = 7.8$ Using tables: $P(X > 10) = 1 - P(X \le 10)$ $= 1 - 0.8352 = 0.1648$	M1 for probability calculation A1 **cao** B1 for mean (SOI) **cao** M1 for probability A1 **cao**	**5**
(v)	Mean no. of items in 50 days = 50 × 1.3 = 65 Using Normal approx. to the Poisson $X \sim N(65, 65)$: $P(X \ge 79.5) = P\left(Z > \dfrac{79.5-65}{\sqrt{65}}\right)$ $= P(Z > 1.799) = 1 - P(Z \le 1.799)$ $= 1 - 0.9641$ $= 0.0359$ (to 3 s.f.)	B1 for Normal approx. (SOI) B1 for continuity corr. M1 for probability A1 **cao**	**4**
			15

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Question 4

(i)	(*A*) If 3 out of 4 are correctly matched then the fourth must also be correct. (*B*) $9k + 8k + 6k + k = 1 \Rightarrow 24k = 1$ $\Rightarrow k = \frac{1}{24}$ (*C*) There are 4! = 24 different arrangements of which just one has all four correctly matched. (All are equally likely), so $P(X = 4) = \frac{1}{24}$ *Or* $P(X = 4) = \frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times 1 = \frac{1}{24}$	E1 M1 for forming equation A1 E1 for 4! arrangements E1 for "just one has all 4 correctly matched" *Or* E1 for $\frac{1}{4}\times p\times q\times r$ [$r = 1$ may be implied] E1 dep. for correct p, q, r	5
(ii)	$E(X) = \Sigma\, r\, P(X = r)$ $= 0 \times \frac{9}{24} + 1 \times \frac{8}{24} + 2 \times \frac{6}{24} + 4 \times \frac{1}{24} = \frac{24}{24} = 1$ $\text{Var}(X) = E(X^2) - [E(X)]^2$ $= 0 \times \frac{9}{24} + 1 \times \frac{8}{24} + 4 \times \frac{6}{24} + 16 \times \frac{1}{24} - 1^2$ $= \frac{48}{24} - 1 = 1$	M1 for E(X) A1 **cao** M1 for $E(X^2)$ A1	4
(iii)	Mean prize money = £100 x 1 = £100 Variance of prize money = 100^2 x 1 = 10000	B1 for mean B1 for variance	2
(iv)	(*A*) P(just one correct in three out of 5 rounds) $= \binom{5}{3} \times \left(\frac{8}{24}\right)^3 \times \left(\frac{16}{24}\right)^2$ $= \frac{40}{243} = 0.1646 = 0.165$ (to 3 s.f.) (*B*) Expected prize money in the five extra rounds = 5 × £1000 x 1 = £5000 So total expected money = £5000 + £400 = £5400	M1 for $\binom{5}{3} \times (8k)^3 \times (1 - 8k)^2$ A1 **cao** M1 for "5000×E(X) + ..." A1 **cao**	4
			15

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General Comments

Overall the performance of candidates was slightly better than in the June 2004 paper. There were fewer very weak scripts and a number of outstanding submissions. The candidates showed, on the whole, a good grasp of the basic methods including accurate and structured solutions.

The stronger candidates scored highly on all the questions with only the final comments in question 1 and the final probability calculation in question 2 causing regular problems. Weaker candidates tended to gain the majority of their marks in calculating Spearman's rank correlation coefficient and carrying out the associated hypothesis test in question 1 and working though standard Normal calculations in question 2.

Most answers were well presented and generally supported by sensible working and explanations.

Comments on Individual Questions

1) **Bivariate data: Spearman's rank correlation: calculation, hypothesis test, comments: comparison of marks of two judges of shops in a retail chain.**

This was a good starting question for most candidates. The first two parts were usually well answered. The final two parts discriminated well between stronger and weaker candidates. It was pleasing to see that the majority of candidates set out their working for the hypothesis test in a clear and logical fashion.

(i) Most candidates demonstrated that they knew how to calculated Spearman's rank correlation coefficient proficiently. There were only occasional arithmetic slips; common errors were forgetting the " 1 – " in the formula and numerical slips in squaring *d*. The weakest candidates attempted calculations based on the difference in the marks, rather than the ranks, for which no credit was given.

(ii) Generally candidates set out their hypotheses and subsequent calculations and explanation very well. However, a prevalent error was to write the alternative hypothesis as a two-tailed test. Most candidates compared the test statistic with the critical value and expressed their conclusion in context. Candidates using a two-tailed test were able to gain all but one of the marks for this part of the question.

(iii) Only the most able candidates gained both marks for their comment. The required answer was that "the background population should be *bivariate Normal*". Credit was then given for the candidate's evidence in discussing whether or not the scatter formed an ellipse, and hence whether or not the product moment correlation coefficient was valid. Often only one mark was gained by discussing the elliptical nature of the scatter, with no mention of bivariate Normality. Weaker candidates missed the point completely, referring only to the linearity of the data.

(iv) This part found most candidates wanting. The modal mark was 1 out of 3. Most candidates did not use ranks, but preferred to compare the performance of shops G and J using marks, often concluding that "shop G was best because it gained the highest total (or average) marks from the two judges". A more subtle analysis was required. Since there was such a discrepancy in the spread of marks of the two judges, then ranks would be better to compare the shops' performance. To gain full marks candidates were expected to compare ranks given to the shops by both judges: shop J came 1st and 3rd, whereas shop G came 5th and 1st. Whilst shops G and J were both awarded 1st place by one of the judges, shop J had a better aggregate ranking than shop G.

2) **Normal distribution: sketch diagram, Normal and binomial calculations: modelling the distribution of lengths of men's trousers**

This probability question turned out to be accessible by even the weakest of candidates. The majority of candidates scored full marks in parts (i), (ii) and (iv). However, only a small minority gained any credit in part (v).

(i) Nearly all candidates gained both marks for the sketch. Occasionally a marked was dropped because of poor labelling.

(ii) Most candidates demonstrated a good understanding of using and applying a Normal probability calculation, with many gaining full marks. Occasional errors were usually in manipulation of the probabilities, e.g. using '0.9192 – 0.7976', which lost the last two marks.

(iii) The binomial calculation was often carried out correctly, but some candidates either misinterpreted this part of the question or omitted it altogether. A large number of candidates failed to gain credit for the assumption, some missing it out completely. Even when attempted, the required answer of 'a random sample from the population' was rarely seen.

(iv) Most candidates knew how to use the Normal distribution 'backwards' and gained all three marks for finding the shortest length for *Extra Long* trousers.

(v) This final part of the question was rarely attempted, presumably because of lack of understanding of what was required. Even when a solution was given, it was rarely correct. Only the strongest candidates were able to express the condition in terms of solving the inequality "$1 - 0.98^n > 0.9$". Even fewer successfully used logarithms, or very occasionally trial and improvement, to find the required value of n (114). A very small number of candidates successfully applied a Poisson approximation (expecting a 'large' *n* with the small *p*).

3) **Poisson distribution: calculations and comments, Normal approximation: modelling the distribution of the number of items of junk mail received daily.**

This question proved a good mark earner for most candidates. The main error was the lack of precision in the answer given for the descriptive part.

(i) Very few candidates scored both marks in identifying two features for which a Poisson distribution would be suitable. The required answers were "uniform average rate of occurrence" and "junk mail is likely to arrive randomly and/or independently". Quite often the second response was not put in context, thus losing the mark available.

(ii) Most candidates successfully explained why the mean was 1.3, and many followed this by concluding correctly that the variance was 1.25. However, there seemed to be much confusion here between a statistical calculation – which was required – rather than a probability calculation – which was condoned. Prevalent errors in calculating the variance included 'forgetting to divide by 100'and 'forgetting to subtract 1.3^2'.

(iii) This was part usually well done, with a correct conclusion that another good reason for using the Poisson distribution as a model was that the sample mean and variance were approximately equal.

(iv) In part (A), nearly all candidates calculated the Poisson probability correctly using the formula. Very rarely were tables used, despite being a perfectly good method.
In part (B), the correctly value of λ (7.8) was identified by nearly everyone, with many going on to gain full marks for this part of the question. However, a disturbingly large minority of candidates interpreted '$P(X > 10)$' as '$1 - P(X \le 9)$' rather than '$1 - P(X \le 10)$', this losing the final two marks.

(v) It was good to see many completely correct responses to the "Normal approximation to the Poisson distribution". Candidates seemed to be well prepared for this type of calculation. Prevalent errors, which have occurred on previous occasions, included the omission of, or incorrect, continuity correction, and incorrect use of the extrapolated variance from part (i) in the Normal approximation.

4) **Discrete random variables: calculations and explanations, expectation and variance: matching pictures with locations.**

Most of this question was successfully attempted by the majority of candidates. However, parts (i) (C) and (iii) (calculation of the variance) proved a pitfall for many.

(i) Reponses to part (A), explaining why P(X = 3) = 0, were often very good, as was the derivation of the constant k in part (B). The modal mark for part (C), using a probability argument to show that P(X = 4) = $\frac{1}{24}$, was 0. Only the most able candidates were able to provide a suitable explanation, usually in terms of a product of probabilities.

(ii) Most candidates were able to score full marks in evaluating the expectation and variance for this discrete random variable. Occasionally some used decimals and lost at least one accuracy mark. On rare occasions candidates forgot to subtract E(X)^2 from E(X)^2.

(iii) Most candidates obtained the correct expectation (£ 100), but failed to find the correct variance (£^{2} 10000), for getting the rule Var(aX) = a^2Var(X).

(iv) Many candidates used the binomial distribution correctly in part (A), but slightly fewer correctly found the expected prize money in all six rounds (£5400). A prevalent error was to forget that the first round did realise a prize of £400 and use the expected value (£100), thus realising a total of £5100. This answer gained the method mark, but not the accuracy mark.